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On Low-Degree Algebraic Points Arising from Morphisms between Generalized Fermat Curves and Their Quotients $\mathcal{F}_{\gamma}^{m}(p)$ and $\mathcal{C}_{\eta,\lambda}^{m}(p)$

Communication Info

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Abstract

In this work, we provide an explicit description of algebraic points of degree at most 3 on the quotient of the Fermat curve $\mathcal{C}_{1,1}^{1}(p)$. We then extend this result via the morphism $\varphi_{r,s}^{\alpha,\beta}$ between quotient Fermat curves $\mathcal{C}_{\eta,\lambda}^{1}(p)$, where $(\eta,\lambda) \in \{1\} \otimes \left\{1, \frac{p-1}{2}, p-2\right\}$. Furthermore, we use the isomorphisms $\Phi_{m,p}$, $\Psi_{m,p}$, and $\Upsilon_{m,p}$ to relate these curves to Fermat-type curves $\mathcal{F}_{\gamma}^{m}(p)$, with $\gamma \in \{1,2\}$. Our approach relies on classical methods as well as fundamental results such as Clifford's theorem and the Riemann--Roch theorem. These tools allow us to describe a basis of the associated linear system and to apply the Abel--Jacobi theorem in order to determine the explicit form of the function associated with a given rational divisor. This ultimately enables us to characterize the structure of these points together with the constraints that define them.

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