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A Variational High-Order Implicit Block Method for the Numerical Solution of Real-World First-Order Dynamical Models

Communication Info

Authors:
Mark I. MODEBEI[1]
Olumide O. OLAIYA [2]

[1,2] National Mathematical Centre, Abuja, Nigeria

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Abstract

We present a variational high-order implicit block method for the numerical solution of first-order ordinary differential equations (ODEs). Derived from a Galerkin variational formulation using polynomial trial functions, the method produces a self-starting, fully implicit block integrator with a compact computational structure. Unlike classical Runge–Kutta and linear multistep schemes, it achieves high accuracy with a formal convergence order of eight and excellent stability properties. Numerical experiments on benchmark problems, including nonlinear dynamical systems, pharmacokinetic models, and semi-discrete PDE systems, confirm the theoretical order and reveal pronounced superconvergence, with observed convergence rate higher than the theoretical order. The variational framework explains the enhanced accuracy and stability. Overall, the proposed scheme offers an efficient, robust, and highly accurate alternative for solving general first-order ODE models in real-world applications [1–5].

References

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