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Twist Families Around Parametric Rank-3 Congruent Numbers: An Experimental Study

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Abstract

We present an experimental investigation of twist families around a set of congruent numbers whose associated elliptic curves have Mordell-Weil rank over 3. Starting from rank-zero elliptic curves of the form

$$E_p: y^2 = x^3 - p^2x$$

previously used to generate congruent numbers via unit-circle parameterizations, we single out five "core" congruent numbers N , for which the curves

$$E_N: y^2 = x^3 - N^2x$$

have rank 3. These cores are interpolated into explicit parametric families $(N_i(t))$, forming a tower of congruent numbers with numerous specializations of positive rank. Around each rank-3 core, we then compute ranks in two natural twist families: the "internal" family

$$E_{N,d}: y^2 = x^3 - dN^2x$$

and the standard congruent number twists

$$E_{Nd}: y^2 = x^3 - (dN)^2x$$

for various square-free integers d . The data systematically exhibit clusters of twists of rank at least 1 and 2, and for each core N , several twists of rank 3 in at least one of the geometries. While no general theorem is proved, the computations suggest that rank-3 congruent numbers arising from unit-circle constructions tend to sit at the center of rich twist families of high rank.

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