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A Mathematical Framework for Root Water Uptake in Unsaturated Soils

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Abstract

This study presents a mathematical model describing water uptake by a single plant root embedded in unsaturated soil, with explicit consideration of the interaction between the root and the surrounding rhizosphere. Building upon established models of root water transport (Landsberg and Fowkes, [1]; Doussan et al [2]; Tournier, [3]), the formulation introduced in Allam et al. [4] extends the classical one-dimensional framework by incorporating both radial and axial hydraulic conductances within the root architecture. Water flow in the rhizosphere is governed by Richards' equation, while internal transport within the root is modeled by a diffusion-type equation, leading to a coupled nonlinear parabolic-elliptic system of partial differential equations.

A rigorous mathematical analysis establishes the well-posedness of the model. Existence and uniqueness of solutions are obtained through a time semi-discretization approach combined with the Schauder-Tychonoff fixed-point theorem. A mixed finite element method is employed to approximate both hydraulic potential and flux in the soil-root system. Numerical experiments demonstrate consistency with theoretical predictions and previously reported physical behavior, supporting the validity of the model.

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