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Asynchronous Exponential Growth In a Nonlinear Age-structured SIR Epidemic Model: Compactness And Irreducibility

Communication Info

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Abstract

This paper presents a rigorous mathematical analysis of a nonlinear age-structured SIR epidemic model. The model is formulated as an abstract Cauchy problem in the L^1 Banach space, and its well-posedness is first established using the theory of C_0-semigroups and Lipschitz continuity of the nonlinear term. The SIR epidemic model is renowned as one of the fundamental frameworks for studying epidemics [1] [2] [3] [4]. The core of our analysis focuses on the asymptotic behavior of the solution semigroup $\mathbf{U}(t)$. We prove that it is eventually compact and irreducible under specific, biologically relevant conditions on the fertility, mortality, and removal rates. By applying the generalized Perron-Frobenius Theorem for positive semigroups, we establish necessary and sufficient conditions for the model to exhibit Asynchronous Exponential Growth (A.E.G.). This result provides a complete characterization of the long-term dynamics of the age-structured epidemic. Numerous researchers have investigated a range of age-structured epidemic models, as documented in references [5].

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