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A linear finite element scheme for the Landau-Lifshitz-Bloch equation

Communication Info

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Abstract

The phenomenological Landau-Lifshitz-Bloch equation suggested by Garanin [3] to describe the precessional motion of spins in ferromagnetic materials takes; when the temperature $\boldsymbol{\theta}$ exceeds the Curie temperature $\boldsymbol{\theta_c}$; the following form:

$$\boldsymbol{u_t = \kappa_1 \Delta u + \gamma u \times \Delta u + \kappa_2 (1 + \mu |u|^2) u}.$$

Where $\boldsymbol{u(t,x) \in \mathbb{R}^3}$ is the spin polarization vector and $\boldsymbol{\gamma}$ is the gyromagnetic ratio. $\times$ and $|\cdot|$ denote respectively the vector product and the Euclidean norm in $\boldsymbol{\mathbb{R}^3}$. $\boldsymbol{\kappa_1}$ and $\boldsymbol{\kappa_2}$ are two damping parameters and $\boldsymbol{\mu = \frac{3\theta}{5(\theta - \theta_c)}}$.

In this communication, we will improve the nonlinear finite element scheme presented in [1] by proposing a linear finite element scheme to the latter equation. We show that our linear scheme converges, without any conditions on the time and space steps, to a weak solution of the LLB equation. Finally, we perform a numerical experiment to compare our linear scheme with a nonlinear scheme already existing in [1].

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