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Preservation of Structural Properties for a Nonlinear 3/2 Mean-Reverting Stochastic Diffusion Process Using θ -Milstein Scheme

Communication Info

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Abstract

This work investigates the stochastic θ -Milstein discretization scheme for the nonlinear mean-reverting diffusion process known as the 3/2 model

$$dX(t) = \alpha(\mu - X(t))dt + \sigma X(t)^{3/2} dW(t),$$

where $\alpha, \mu, \sigma > 0$. Extending the positivity-preserving analysis of the Cox-Ingersoll-Ross model [1], we prove that the scheme preserves the non-negativity of the exact solution and retains the essential mean-reversion property. Despite the challenges posed by the non-Lipschitz diffusion term, we establish that the method achieves a strong convergence rate of 1, consistent with findings for the Heston 3/2 model [2]. Importantly, the scheme also captures the long-term variance accurately. Numerical experiments validate these theoretical findings and show a clear advantage of the θ -Milstein scheme over the standard explicit Euler-type method [3], which often fails to preserve positivity.

References

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