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## Second-order dynamical systems with time-dependent viscous damping coefficient

### Communication Info

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- (1) second order dynamical systems ;
- (2) constrained minimization;
- (3) gradient systems.

### Abstract

This work studies the asymptotic behavior of the following second-order dynamical system, for  $\alpha > 0$ :  
for  $\alpha > 0 : \ddot{x}(t) + (\alpha/t)\dot{x}(t) + \nabla\varphi(x(t)) + \beta(t)\nabla\psi(x(t)) = 0, t > t_0 > 0$ . |where  $\varphi, \psi: \mathcal{H} \rightarrow \mathbb{R}$  are convex and smooth functions defined on a real Hilbert space  $\mathcal{H}$ ,  $\gamma > 0$ , and  $\beta$  is a time-dependent function controlling the penalty term.

Such systems arise naturally in optimization, mechanics, and control, where friction is not always constant but can evolve during the process. Earlier research has shown how first-order gradient systems with penalty terms and second-order inertial systems with constant damping converge to equilibrium under suitable conditions. Here, we go one step further by allowing the damping effect itself to vary with time, which provides access to a broader class of inertial optimization algorithms. We establish conditions ensuring that the system still converges weakly to equilibrium points and show how the specific evolution of the damping term influences this convergence.

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### References

- [1] Boț, Radu Ioan and Ernő Robert Csetnek. "Second-order dynamical systems associated to variational inequalities." *Applicable Analysis* 96 (2015): 799 - 809.
- [2] Hedy Attouch, Marc-Olivier Czarnecki, "Asymptotic behavior of coupled dynamical systems with multiscale aspects", *Journal of Differential Equations*, Volume 248, Issue 6, (2010): 1315-1344.