ICRAMCS 2026

THE EIGHTH EDITION OF THE INTERNATIONAL CONFERENCE ON RESEARCH IN APPLIED MATHEMATICS AND COMPUTER SCIENCE

April 23-24-25, 2026 | Marrakech, Morocco

A Hyperbolic Conservation-Law Model for Solidification and Settling: Finite-Volume Simumation, Shocks/Rarefactions and Validation by Characteristics.

Communication Info

Authors:

Morad LAKHSSASSI[1,2]

Youssef JOUNDY [1]

Ahmed TAIK [1]

[1] Faculté des Sciences et Techniques, Mohammedia, Morocco

[2] Université Internationale de Casablanca, Casablanca, Morocco

Keywords:

(1) Hyperbolic PDE

(2) Shocks and rarefactions

(3) Method of characteristics

(4) Finite volume method

(5) Crystallization and settling

Abstract

We present a 1D conservation-law model for the evolution of the solid fraction $p_s(x,t) \in [0,1]$ (porosity $p_l = 1 - p_s$), in a closed vertical column, combining production by solidification and transport by settling. The model reads [1] :

$$\partial_t p_s + \partial_x f(p_s) = A.\, q_s(x), \quad f(p_s) = v_s(p_s)\, p_s$$

where $q_s(x)$ is a prescribed depth-dependent solidification rate and A measures the relative importance of advective transport versus solidification. Settling is described by a Stokes-based, Richardson–Zaki hindered-settling law [2], leading to $v_s(p_s) = (1 - p_s)^5$ and the nonlinear flux $f(p_s) = p_s(1 - p_s)^5$, whose non-monotonicity leads to shock/rarefaction structures typical of hyperbolic PDEs [3]. The numerical solution is obtained using an implicit conservative finite-volume Lax–Friedrichs scheme [4] solved by Newton–Raphson iterations. Two dedicated corrections are introduced: one to accurately capture the moving sedimentation front, and another to eliminate spurious fluxes in fully solidified cells. The robustness of the proposed approach is demonstrated through a hierarchical validation strategy. This strategy ranges from limiting analytical configurations, where transport and/or solidification mechanisms are selectively deactivated, to characteristic-based verification. Quantitative comparisons are performed for purely advective cases, while qualitative checks guided by characteristic analysis are carried out for fully coupled configurations [5].

References

[1] M. Lakhssassi, B. Guy, E. Touboul, and J.-Y. Cottin, Bimodal distribution of the products of a magmatic chamber: modelling by fractional crystallization and coupling of the chemical exchanges with the differential magma/solid transport. Comptes Rendus de l'Académie des Sciences, geosciences 342, 9, 2010, 701-709.

[2] J. F. Richardson and W. N. Zaki. Sedimentation and uidization. part i. Trans. Inst. Chem. Eng., 32, 1954, 35-53.

[3] P. D. Lax. Hyperbolic systems of conservation laws, ii. Comm. Pure Appl. Math., 10, 1957, 537-566.

[4] R. J. LeVeque. Large time step shock-capturing techniques for scalar conservation laws. SIAM J. Numer. Anal., 19, 1982, 1091-1109.

[5] Godlewski, E. & Raviart, P.-A, Numerical Approximation of Hyperbolic Systems of Conservation Laws. Applied Mathematical Sciences, Vol. 118, 1996, 509 p.