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Existence and Uniqueness of Renormalized Solutions for a p(x)-Elliptic Non-Coercive Problem Under Neumann Boundary Conditions

Communication Info

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Abstract

Let $\Omega \subset \mathrm{R}$ $(\mathrm{n} \geq 1)$, be a bounded domain with smooth $\partial\Omega$ with Lipschitz boundary $\partial\Omega$,

This paper addresses the following quasilinear elliptic problem with Neumann boundary condition:

$$\begin{cases} -\boldsymbol{div}\left(\dfrac{\boldsymbol{a}(\boldsymbol{x},|\boldsymbol{\nabla u}|)\boldsymbol{\nabla u}}{(\mathbf{1}+|\boldsymbol{u}|)^{\boldsymbol{\theta}(\boldsymbol{x})}}\right)+|\boldsymbol{u}|^{\boldsymbol{p}(\boldsymbol{x})-\mathbf{2}}\boldsymbol{u}=\boldsymbol{f}-\boldsymbol{div}(\boldsymbol{F})\ \boldsymbol{in}\ \boldsymbol{\Omega}, \\ \left(\dfrac{\boldsymbol{a}(\boldsymbol{x},|\boldsymbol{\nabla u}|)\boldsymbol{\nabla u}}{(\mathbf{1}+|\boldsymbol{u}|)^{\boldsymbol{\theta}(\boldsymbol{x})}}-\boldsymbol{F}\right).\boldsymbol{\nu}=\boldsymbol{g}\ \boldsymbol{on}\ \boldsymbol{\partial\Omega}\,, \end{cases}$$

where ν is the outer unit normal vector on $\partial\Omega$. The exponents $p(x), \theta(x)$ are continuous functions. The data f and g belong to $L^1(\Omega)$ and $L^1(\partial\Omega)$ respectively, $F \in (L^1(\Omega))^N$.

Our aim is to prove the existence and uniqueness of renormalized solutions for the quasilinear elliptic problem above.

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