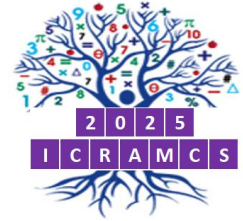


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Existence and Uniqueness of Renormalized Solutions for a $p(x)$ -Elliptic Non-Coercive Problem Under Neumann Boundary Conditions

Communication Info

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Abstract

Let $\Omega \subset \mathbb{R}^n$ ($n \geq 1$), be a bounded domain with smooth $\partial\Omega$ with Lipschitz boundary $\partial\Omega$,

This paper addresses the following quasilinear elliptic problem with Neumann boundary condition:

$$\begin{cases} -\operatorname{div} \left(\frac{a(x, |\nabla u|) \nabla u}{(1 + |u|)^{\theta(x)}} \right) + |u|^{p(x)-2} u = f - \operatorname{div}(F) \text{ in } \Omega, \\ \left(\frac{a(x, |\nabla u|) \nabla u}{(1 + |u|)^{\theta(x)}} - F \right) \cdot \nu = g \text{ on } \partial\Omega, \end{cases}$$

where ν is the outer unit normal vector on $\partial\Omega$. The exponents $p(x), \theta(x)$ are continuous functions. The data f and g belong to $L^1(\Omega)$ and $L^1(\partial\Omega)$ respectively, $F \in (L^1(\Omega))^N$.

Our aim is to prove the existence and uniqueness of renormalized solutions for the quasilinear elliptic problem above.

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