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Hopf Bifurcation in a Delayed Predator–Prey Model with Reproduction Lag and Toxic

Communication Info

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Abstract

We study a predator–prey model with two time delays describing the interactions between sea urchins (prey) and crabs (predators). The model incorporates the Allee effect on prey growth as well as toxic effects induced by prey consumption on predators. Two delays are introduced to represent the delayed defense mechanisms of the prey and the reproduction time of the predators. The analysis focuses on the local stability of the coexistence equilibrium and the occurrence of Hopf bifurcation with respect to the delays. Conditions ensuring the emergence of periodic solutions are derived when the delays pass certain critical values. By applying the center manifold theory and normal form method, the direction of the Hopf bifurcation and the stability of the bifurcating periodic solutions are determined.

Numerical simulations are provided to support the theoretical findings and to illustrate the impact of time delays on the system dynamics.

References

[1] Hafdane M., Agmour I., El Foutayeni Y. Study of Hopf bifurcation of delayed tritrophic system: dinoflagellates, mussels, and crabs. Mathematical Modeling and Computing, Vol. 10, No. 1, pp. 66–79 (2023)
[2] Akkocaoglu H, Merdan H, Çelik C (2013) Hopf bifurcation analysis of a general non-linear differential equation with delay. J Comput Appl Math 237:565–575.
[3] Hassard BD, Kazarinoff ND, Wan YH (1981) Theory and applications of hopf bifurcation. Cambridge University Press, Cambridge.
[4] El Foutayeni Y, Khaladi M (2016) Equilibrium Points and Their Stability Properties of a Multiple Delays Model. Differ Equa Dyn Syst Springer. https://doi.org/10.1007/s12591-016- 0321-y.
[5] Ruan S, Wei J (2001) On the zeros of a third degree exponential polynomial with applications to a delayed model for the control of testosterone secretion. IMA J Math Appl Med Biol 18(18):41–52